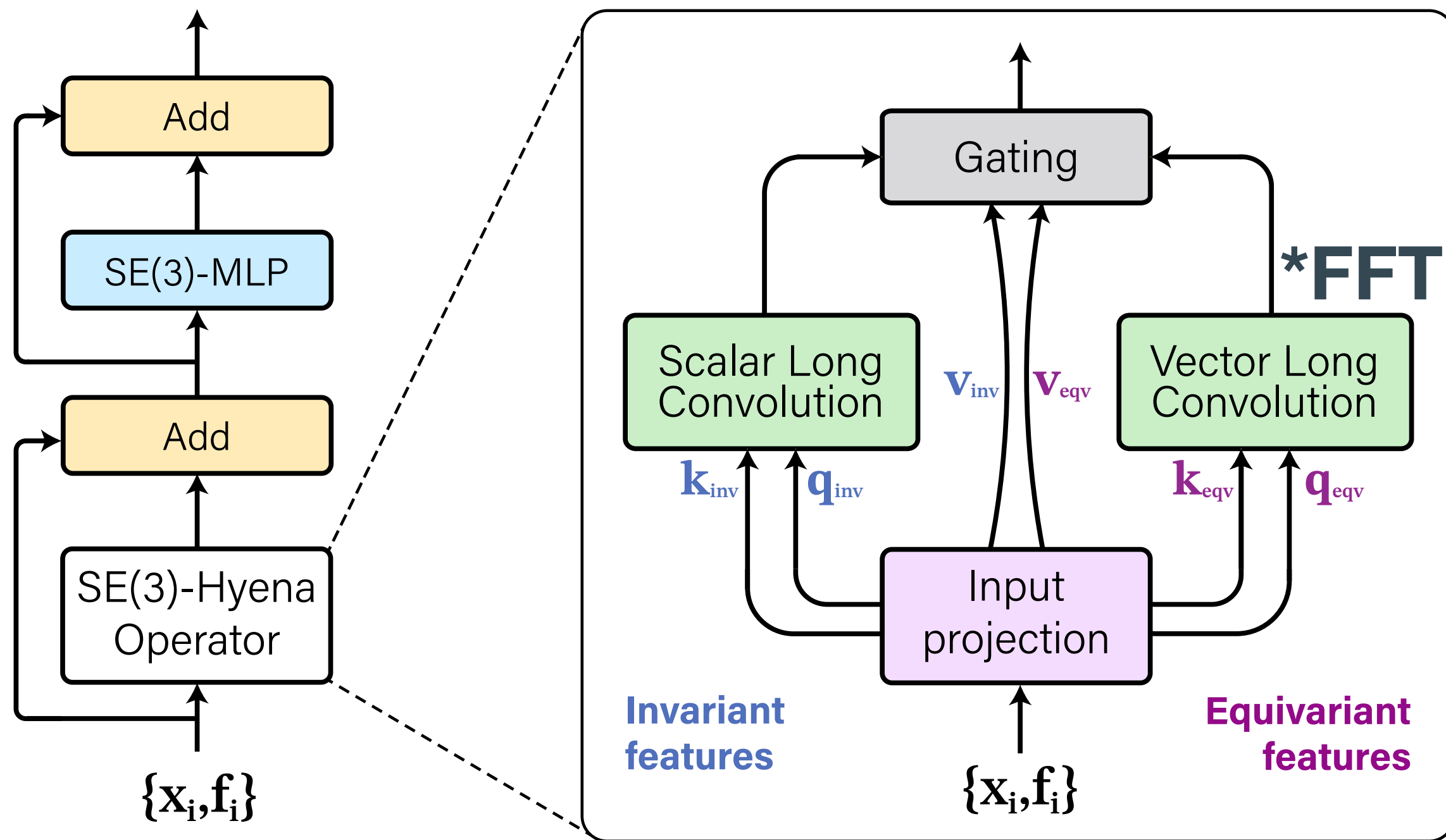
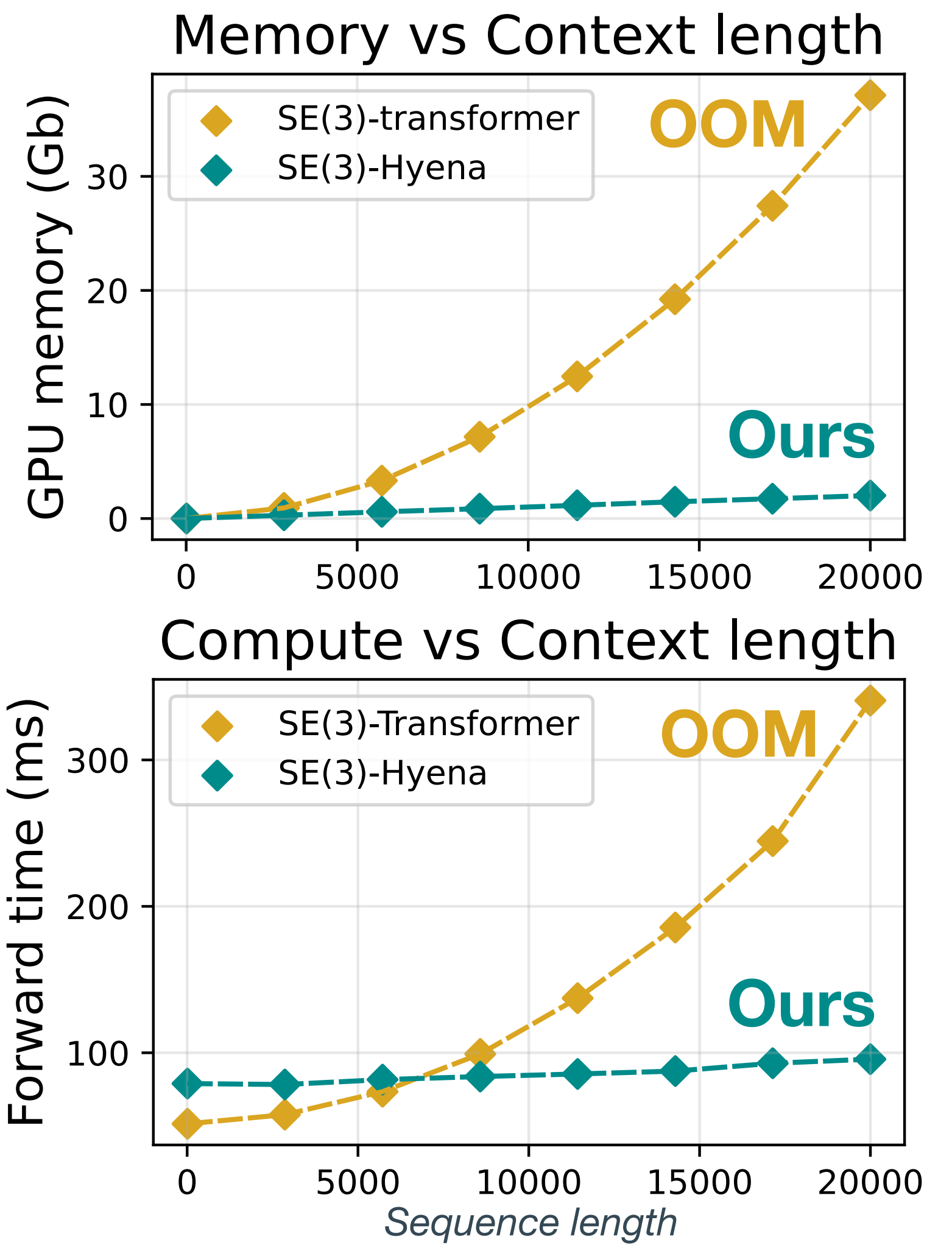


SE(3)-Hyena Operator for Scalable Equivariant Learning

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- Processing global geometry is crucial in biology, chemistry, materials, graphics, etc..
- Quadratic self-attention is prohibitively expensive
- We develop **first long-convolutional equivariant network** with $O(N \log N)$ complexity



Equivariantly process up to 3.5M tokens with global (all-to-all) context on a single A10 GPU.

Input projection:

Input features:

$\{\mathbf{x}_i, \mathbf{f}_i\}_{i=1}^N$ \mathbf{x} : equivariant vector tokens
 \mathbf{f} : invariant scalar tokens

Equiv. MLP gives data-controlled filter:

$\mathbf{z}_i^{eqv}, \mathbf{z}_i^{inv} = \phi(\mathbf{x}_i, \mathbf{f}_i)$

$[\mathbf{q}_i^{eqv}, \mathbf{k}_i^{eqv}, \mathbf{v}_i^{eqv}]$ query, key, value for each token

**Simple implementation:
50 lines of code**

Equivariant and invariant long convolution:

Invariant scalar long convolution:

$$(\mathbf{q} \circledast \mathbf{k})_i = \sum_{j=1}^N q_i k_{j-i} = (\mathbf{F}^H \Lambda_k \mathbf{F} \mathbf{q})_i$$

$O(N \log N)$

Scalar long convolution

Equivariant vector long convolution:

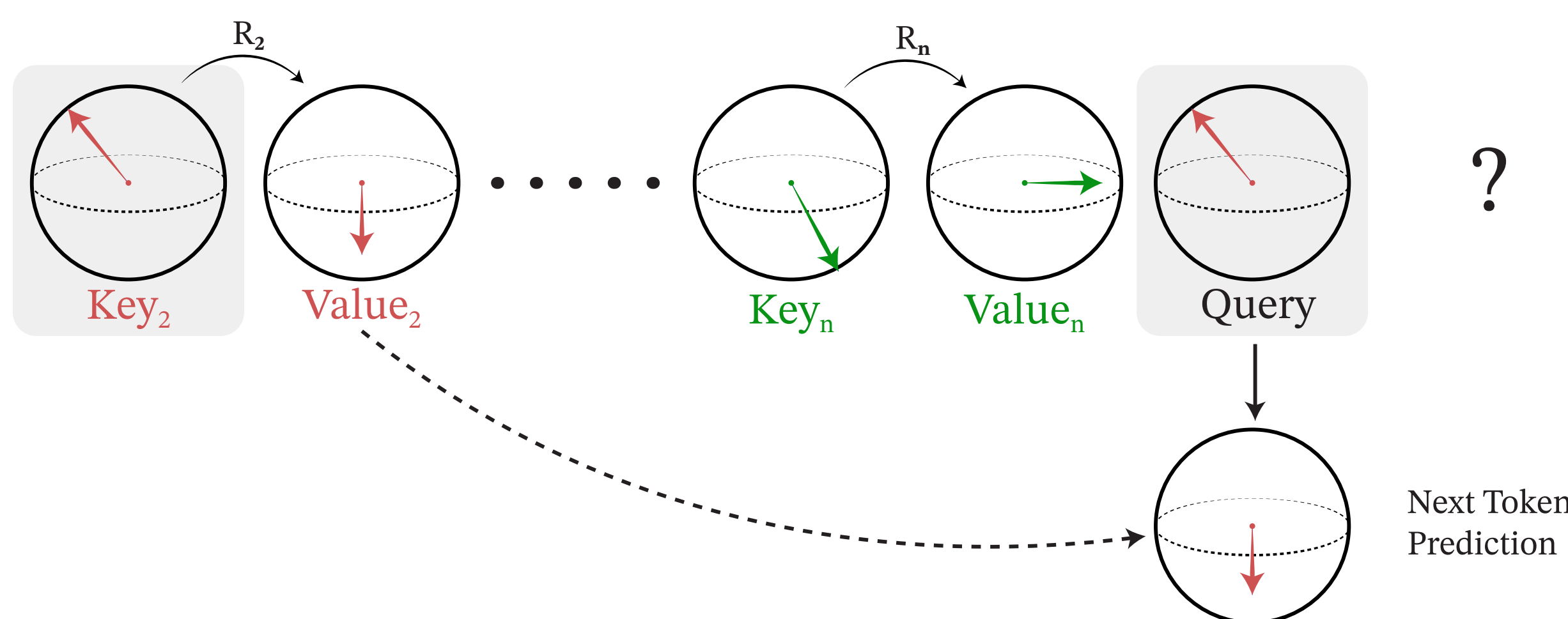
$$(\vec{\mathbf{q}} \circledast_{\times} \vec{\mathbf{k}})_i = \sum_{j=1}^N \vec{\mathbf{q}}_i \times \vec{\mathbf{k}}_{j-i}$$

$$(\vec{\mathbf{q}} \circledast_{\times} \vec{\mathbf{k}})_i[l] = \varepsilon_{lhp} \sum_{j=1}^N \vec{\mathbf{q}}_i[h] \times \vec{\mathbf{k}}_{j-i}[p]$$

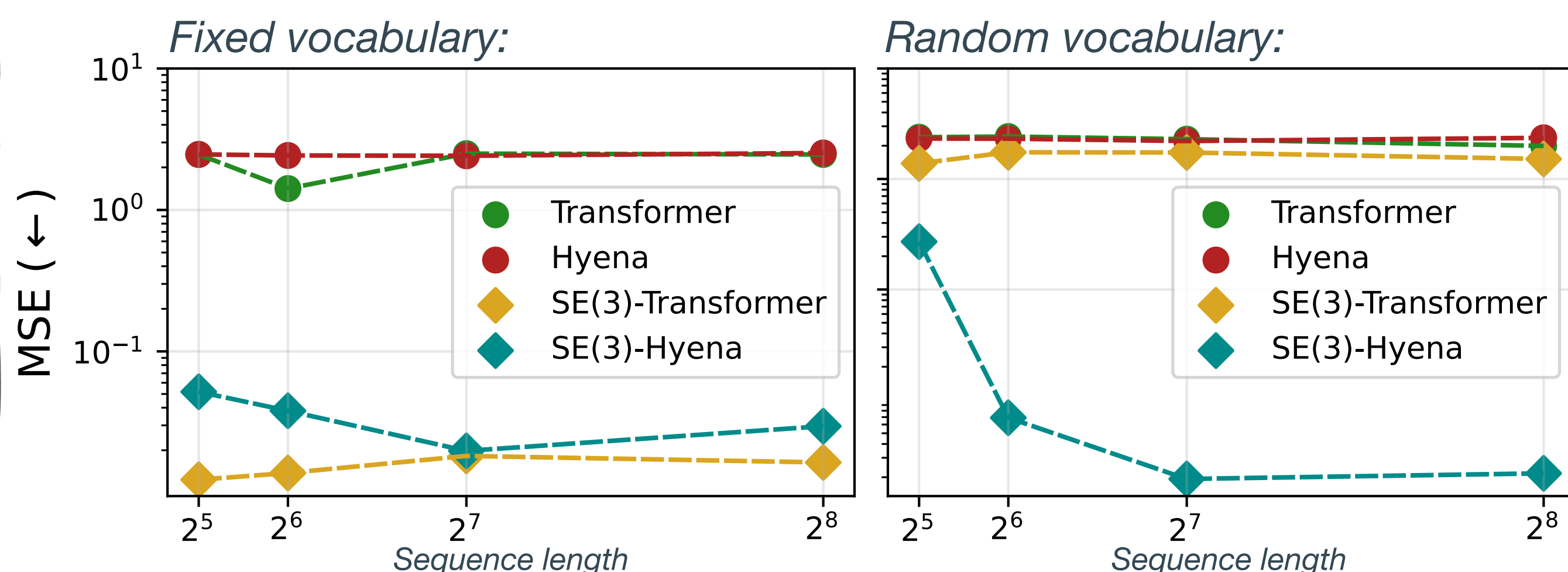
$$(\vec{\mathbf{q}}[h] \circledast_{\times} \vec{\mathbf{k}}[h])_i$$

FFT

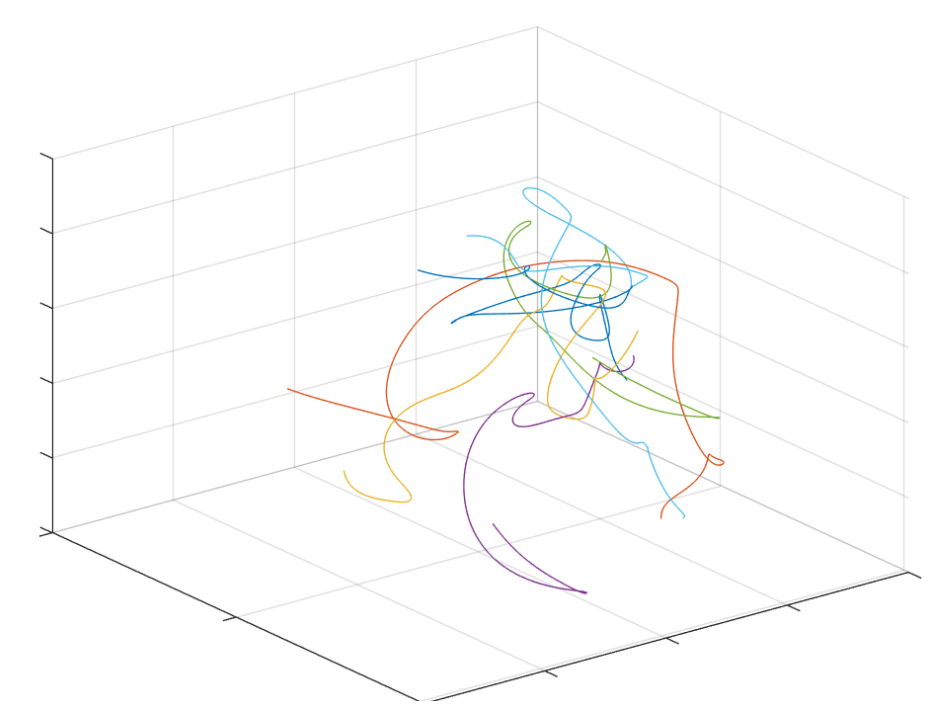
Equivariant associative recall:



On-par or better than self-attention



Equivariant n-body system:



Method	MSE
Linear	0.0322
Transformer	0.0163
Hyena	0.0150
SE(3)-Transformer	0.0019
SE(3)-Hyena	0.0018

